Torsional Pendulum Video Writeup

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This study investigates the oscillation period of a system, comparing theoretically calculated values against experimentally measured periods. The results demonstrate how variations in wire diameter and moment of inertia correlate with changes in oscillation period, highlighting both the agreement and discrepancies between theoretical predictions and practical observations.

I. Introduction

Telling time is a crucial part of our daily lives. From alarms waking us up to the start of a class or meeting, knowing the time helps structure our entire day. This dependence on timekeeping isn't new—people have always needed a way to track time. But a few hundred years ago, without digital clocks or modern technology, how did they do it?

The answer lies in a simple physical phenomenon known as oscillatory motion. To create oscillatory motion in clocks, there are two main approaches: the familiar swinging pendulum, or the focus of this project...torsional springs.

In a torsion pendulum clock, the base rotates back and forth with a regular period. This rotational oscillatory motion is transferred into the clock's gears through an escapement mechanism, which moves with each complete oscillation. That's what keeps the time consistent and precise.

To measure time accurately with this type of motion, we need to calculate the period of torsional oscillation. Equation 1 shows how to calculate the period of a torsional spring. The two dependencies are the moment of inertia (I) and the constant spring force (k).

$$T = 2\pi\sqrt{\frac{I}{k}} \tag{1}$$

To calculate the moment of inertia, Fusion 360 moment of inertia calculator was used. To calculate the constant spring force, the following equation was used.

$$K = \frac{GJ}{L} \tag{2}$$

The shear modulus, G, is given for a steel material to be 80 GPa. The length of the wire, L, was determined in the design to be 0.15 m. The polar moment of inertia can be calculated for a circular rod with the following equation:

$$J = \frac{\pi d^4}{32} \tag{3}$$

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II. Methods

The experimental setup consisted of a custom-designed test stand, two distinct rotating masses, suspension wires of varying diameters, and fastening components. All structural components (test stand, rotating masses, and spacer block) were fabricated using 3D printing technology.

The test stand (Figure 1) was designed to securely suspend a torsional pendulum assembly. It provided anchor points for the suspension wire at the top and sufficient clearance for the rotating mass to oscillate freely below.

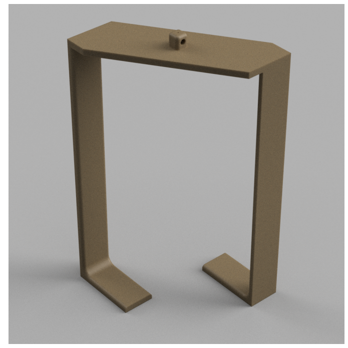

Fig. 1. CAD rendering of the 3D printed test stand used to suspend the torsional pendulum.

Two rotating masses, designated Mass One (Figure 2) and Mass Two (Figure 3), were designed and 3D printed. They were intentionally designed to possess different moments of inertia to investigate its effect on the oscillation period. Each mass included a central bore for the suspension wire and integrated set screws to rigidly fix the wire's position relative to the mass.

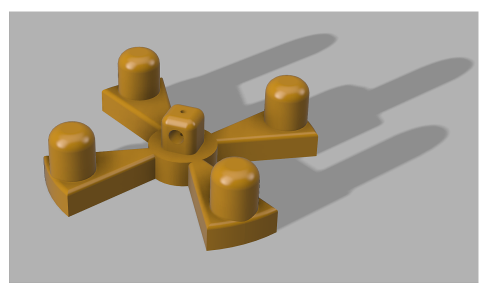

Fig. 2. CAD rendering of Rotating Mass One.

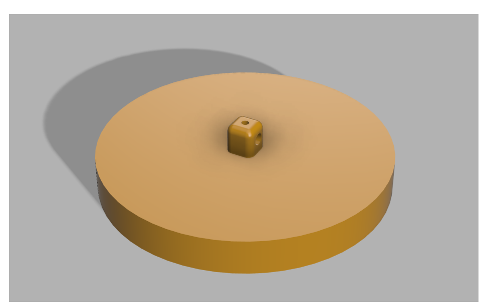

Fig. 3. CAD rendering of Rotating Mass Two.

Suspension wires of four different nominal diameters (0.2 mm, 0.3 mm, 0.4 mm, and 0.5 mm) were used as the torsional springs. Simple set screws were employed both at the top anchor point on the test stand and within the rotating masses to secure the ends of the wire. A 3D printed spacer block was utilized during setup to ensure a consistent active length of the suspension wire between the top anchor and the point of attachment on the rotating mass for all trials.

For each experimental run, a specific diameter wire was selected and cut to length. The wire was first secured at the upper anchor point of the test stand using a set screw. The 3D printed spacer block was then used to define the attachment point for the rotating mass, thereby setting a consistent active wire length. The chosen rotating mass (either Mass One or Mass Two) was then carefully threaded onto the free end of the wire until it reached the position defined by the specific depth within the mass's bore. The set screws on the rotating mass were then tightened to firmly secure the wire.

Once assembled, the torsional pendulum was set into oscillation by applying a small initial angular displacement to the mass and releasing it gently. The period of oscillation was determined by measuring the time elapsed for one or more complete cycles. Specifically, start and stop times were recorded for distinct oscillation events, as detailed in Tables 1 and 2. This process was repeated for three trials for each combination of rotating mass and wire diameter to ensure repeatability and allow for the calculation of an average period. Measurements were systematically conducted for both rotating masses using all four wire diameters.

TABLE I
PERIOD MEASUREMENTS FOR ROTATING MASS ONE WITH VARYING WIRE DIAMETERS.

Wire Diameter (mm)	Start (s)	Stop (s)	Measured Period (s)
0.2	3.96	5.12	1.160
	6.51	7.64	1.130
	8.95	10.31	1.360
0.3	2.53	3.07	0.540
	3.58	4.09	0.510
	6.18	6.69	0.510
0.4	0.72	0.99	0.270
	1.29	1.57	0.280
	2.42	2.71	0.290
0.5	1.07	1.26	0.190
	1.45	1.62	0.170
	1.81	2.00	0.190

TABLE II
PERIOD MEASUREMENTS FOR ROTATING MASS TWO WITH VARYING WIRE DIAMETERS.

Wire Diameter (mm)	Start (s)	Stop (s)	Measured Period (s)
0.2	2.35	4.64	2.290
	7.08	9.43	2.350
	11.75	14.18	2.430
0.3	1.01	2.33	1.320
	4.95	6.36	1.410
	8.98	10.39	1.410
0.4	2.82	3.44	0.620
	4.79	5.39	0.600
	6.17	6.78	0.610
0.5	1.98	2.39	0.410
	3.28	3.73	0.450
	4.20	4.63	0.430

TABLE III
CALCULATED AND AVERAGE EXPERIMENTAL PERIODS FOR ROTATING MASS ONE WITH MOMENT OF INERTIA = $3.76 \times 10^{-6}\ kg \cdot m^2$

Wire Diameter (m)	Calculated Period (s)	Average Exp. Period (s)
2.0×10^{-3}	1.339	1.217
3.0×10^{-3}	0.595	0.520
4.0×10^{-3}	0.335	0.280
5.0×10^{-3}	0.214	0.183

TABLE IV
CALCULATED AND AVERAGE EXPERIMENTAL PERIODS FOR ROTATING MASS TWO WITH MOMENT OF INERTIA = $9.40 \times 10^{-6}\ kg \cdot m^2$

Wire Diameter (m)	Calculated Period (s)	Average Exp. Period (s)
2.0×10^{-3}	2.117	2.357
3.0×10^{-3}	0.941	1.380
4.0×10^{-3}	0.529	0.610
5.0×10^{-3}	0.339	0.430

III. CONCLUSION

The data collected in Tables 1 and 2 clearly demonstrate an inverse relationship between the diameter of the suspension wire and the period of oscillation for both rotating masses. As the wire diameter increased from 0.2 mm to 0.5 mm, the measured period decreased substantially. This observation aligns with the theoretical understanding that a thicker wire provides greater torsional stiffness (a larger spring constant, k), leading to faster oscillations and thus a shorter period. Furthermore, comparing the periods for the same wire diameter between Mass One (Table 1) and Mass Two (Table 2) reveals consistently longer periods for Mass Two, strongly suggesting that Mass Two possesses a larger moment of inertia (I) than Mass One.

Quantitative discrepancies exist between the calculated and average experimental periods. While some pairings show reasonable agreement (within 10-20 percent), others exhibit more significant differences (up to 30 percent or more, notably for Mass Two with the 0.3mm and 0.5mm wires). These deviations could stem from several sources, including uncertainties in the measurement of the wire diameter, potential inaccuracies in the assumed values for the moments of inertia, variations in the material properties (Shear Modulus) of the wire from standard values, or experimental factors not accounted for in the simple theoretical model, such as damping or slight imperfections in the setup. Despite these quantitative differences, the experiment successfully illustrates the dependence of the torsional oscillation period on both the stiffness of the suspension wire and the moment of inertia of the rotating body.