# Lab 8: Spectrum of the Hydrogen Atom

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**This experiment investigates the spectrum of hydrogen by measuring the emitted wavelengths from a hydrogen gas discharge lamp using a diffraction grating spectrometer. The wavelengths were analyzed to understand the quantum energy transitions in the hydrogen atom, which are critical to atomic structure and identification. Using the Balmer and Bohr models, we predicted the wavelengths and compared them to experimental measurements. Despite challenges with alignment and measurement precision, the experimental results were consistent with theoretical expectations. The study demonstrates how spectroscopic techniques can be used to probe atomic structure and provides insights into the impact of measurement uncertainties and experimental errors.**

## I. Introduction

In this lab an experiment will be conducted to measure the wavelength emitted from a gas discharge lamp to calculate the spectrum of the hydrogen atom.

The spectrum of hydrogen is super important to the understanding of quantized energy levels of atoms. The transition of electrons from one level to another can be observed through discrete lines of emitted light when electrons transition between energy levels. With this an understanding of the structure of atom can be deduced, which is greatly helpful in identification of atoms. Hydrogen is one of the most abundant atom in our universe, so enabling the identification well studying celestial objects better build on the collective scientific understanding of the universe.

Observing discrete light spectrum from a hydrogen gas lamp, using a diffraction grating spectrometer, the wavelength of the emitted light can be measured. Below in the methods is the initial derivations and the procedure followed for this experiment.

## II. Methods: Derivation

Gases with a large applied voltage undergo dielectric breakdown and emit bright light. This bright light can be examined with a spectrometer, and used to determine the gas that is present. For hydrogen gas it will only emit four visible lines during dielectric breakdown: red, green, blue and violet.

To predict the wavelengths that will be present when observing the emitted light of hydrogen, a general formula was empirically found by J.J. Balmer. J.J. Balmer formula is what follows:

$$\frac{1}{\lambda} = R(\frac{1}{n_f^2} - \frac{1}{n_i^2}) \tag{1}$$

In the above $n_f and n_i$ are both integer value of the initial and final shell where electrons are present and R is the Rydberg constant. R = 1.0974 $x 10^7 m^{-1}$.

There was a major flaw in the scientific communities understanding of the spectrum of hydrogen and other gases. As the mentioned the formula 1 was derived empirically, it created a gap between fundamental explanation grounded in classical physics and what was seen through experiments.

Then in 1913, Niels Borh suggested a new theory of atoms. This new theory proposed that electrons could only exist in certain energy states and thus would "jump" between them in discrete values. Subsequently, from a "jump", electrons would emit light to conserve energy. This light emitted would also have to be in discrete values. Therefore having the same fixed set of wavelengths(colors). Thus the following formula was derived.

$$E_n = -(\frac{me^4}{8\epsilon_0^2 h^2})\frac{1}{n^2} \tag{2}$$

Taking into account the transition from initial level $n_i$ and the final level $n_f$, plus the energy conserved during the process, the following relationship can be found between Borh's and Balmer's formulea.

$$\frac{1}{\lambda} = \frac{\Delta E}{hc} = \frac{me^4}{8\epsilon_0^2 h^2}(\frac{1}{n_f^2} - \frac{1}{n_i^2}) = R(\frac{1}{n_f^2} - \frac{1}{n_i^2}) \tag{3}$$

## III. Methods: Experimental Set Up

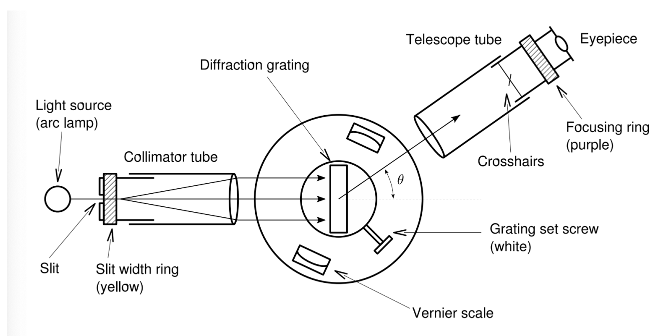


Fig. 1. Diagram of spectrometer used during the experiment.

The figure above, Figure 1, shows the equipment being used in this experiment. The process light goes from initial source to eyepiece is as what follows. The light passes through the collimator tube, which collects light from the source and ensures only parallel light rays are exited. The light then goes through a diffraction grating, getting diffracted allowing a human to see it through the eyepiece as they rotate the telescope tube by utilizing the turntable. To read the angle, $\theta$, there is a vernier scale that allows for precise measurements.

The diffraction grating is a slab of material with a large number of small parallel slits. Each slit diffracts the beam and the beam will then act as a new beam of light with a different angle of emission than before. To see this Figure 2 shows how the grating works.

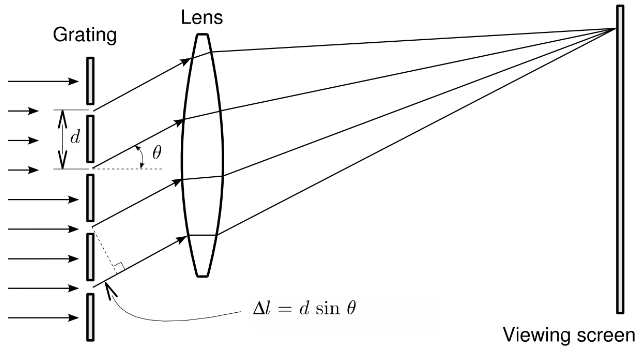


Fig. 2. Diagram of diffraction grading used in this experiment.

## IV. Methods: Procedure

The initial part of the procedure relies on setting up the equipment and the spectrometer as depicted in Figure 1. First the slit was adjusted to be about half way and the grating was removed from its holder. Then at a straight through position, the focus knob was turned until through the eyepiece a sharp image of the slit was present.

Next the tabletop was adjusted so that the zero mark on the Vernier scale was aligned with the $180^o$ mark from the outer scale. To do this the rotation was unlocked and then relocked. Finally the grating was placed back into the holder and the measurements were ready to be taken.

Before angular measurements of the hydrogen spectrum could be measured the grating lattice constant had to be found. To do this a helium discharge lamp was used with a known wavelength of $\lambda = 5.8756x10^{-7}m$. The helium lamp was lined up with the spectrometer and turned on. Coverings were put around the spectrometer to ensure no surrounding light would interfere with measurements. The telescope tube was rotated until the first order yellow line was found on both sides. The crosshair inside the telescope were lined up to the middle of the visible lines and the angle was noted. The average of both numbers were used when calculating the grating lattice constant.

With the grating lattice constant found, now the spectrum of hydrogen could be found. The helium lamp was turned off and the hydrogen lamp was turned on and placed before the spectrometer. Again the telescope was rotated and lined up to the first and second order red, green, blue, and purple lines from both sides.

## V. Analysis

TABLE I
Degrees and Combined Values for the angle, $\theta_{yellow}$, of the first-order yellow line from the helium lamp.

| Description | Degrees | Combined |
|---|---|---|
| $\theta_{left}$ | 20° 51' | 20.850 |
| $\theta_{right}$ | 20° 51' | 20.850 |
| $\theta_{avg}$ | 20° 51' | 20.850 |

The above table shows the measurements found from the first order yellow line on the helium spectrograph. As we know the wavelength of helium to be $5.8756x10^{-7}$, the lattice constant can be found with the following equation.

$$d = \frac{m\lambda}{sin\theta} \quad (4)$$

The lattice constant was found to be, d = $1.6508x10^{-6}$.

The table below represents the values found for each color and order that was seen, with the average angle and $n_i$ value. During the experiment the measurement scale got misaligned which caused an offset of 9 degrees, 56.5 minutes. This offset was found when doing the calculations the final values of $n_i$ and only present in our measurements with the hydrogen lamp. It was determined after discussion with lab partners that correcting this offset was appropriate due to maligned scale present throughout the entirety of the hydrogen lamp part of experiment, the new center of the scale was determined to be 189.833 degrees, no 180 degrees that was originally thought.

TABLE II
Experimental data for the different colors and their corresponding average angles and $n_i$ values.

| Color (Order) | Average Angle (°) | $n_i$ |
|---|---|---|
| Blue (1) | 15.241667 | 5.003083 ± 0.091268 |
| Cyan (1) | 17.108333 | 4.006698 ± 0.037084 |
| Red (1) | 23.400000 | 3.002063 ± 0.008218 |
| Purple (2) | 29.791667 | 6.006417 ± 0.079538 |
| Blue (2) | 31.725000 | 5.001556 ± 0.040184 |
| Cyan (2) | 36.091667 | 3.999338 ± 0.015553 |
| Purple (3) | 48.200000 | 5.999825 ± 0.040569 |
| Red (2) | 52.683333 | 2.999697 ± 0.002701 |

A little note about the uncertainty in measurements. The uncertainty was calculated by finding the distance between the finite line width, which turned out to be a difference of 13 minutes. This value was then used in propagating errors to the final values of $n_i$.

## VI. Discussion

All the questions asked in the lab manual during procedure are answered as follows from ordered presented. During the helium lamp setup that was used to determine the lattice constant, the amount of lines ($mm^{-1}$) that the lattice constant corresponds to was found to be 605.76 $mm^{-1}$ which closely matches the value present on the grate, which was $600mm^{-1}$. Looking for the second and third order yellow, the second order was visible well the third was not visible. During the observation of the hydrogen spectrum the amount of orders that could be seen was three orders out. To find the total number of orders seen the red lines were counted as we rotated the telescope until no further red lines could be seen.

1/2. The final results for the initial atomic shell occupied by the excited valence electron, $n_i$, were not integers, which makes sense for the accuracy of this lab. Taken into account the error that was present with each measurement, each $n_i$ can be reasonably seen as integers. All measurements of $n_i$ are within error amount of being integers. These integer values are consistent with the predictions of the Balmer and Bohr formulae.

TABLE III
EXPERIMENTAL DATA FOR $n_i$ VALUES WITH CORRESPONDING PREDICTED VALUES BASED ON BALMER AND BOHR FORMULAE.

| Color (Order) | Experimental $n_i$ | Predicted $n_i$ |
|---|---|---|
| Blue (1) | 5.003083 $\pm$ 0.091268 | 5 |
| Cyan (1) | 4.006698 $\pm$ 0.037084 | 4 |
| Red (1) | 3.002063 $\pm$ 0.008218 | 3 |
| Purple (2) | 6.006417 $\pm$ 0.079538 | 6 |
| Blue (2) | 5.001556 $\pm$ 0.040184 | 5 |
| Cyan (2) | 3.999338 $\pm$ 0.015553 | 4 |
| Purple (3) | 5.999825 $\pm$ 0.040569 | 6 |
| Red (2) | 2.999697 $\pm$ 0.002701 | 3 |

3. Order 3 appears to give more consistent and precise $n_i$ values, especially for the purple color. The uncertainties here are relatively small compared to order 1 and order 2, suggesting that this order might be more reliable for your measurements. Order 1 and order 2 have some variation in their $n_i$ values, and their uncertainties seem larger, indicating that these orders might not be as accurate as order 3.

4. The color that jumped from the lowest shells was Red. This makes sense as red is among the lowest visible color on the light spectrum. This means that the smaller energy that is required to excite a lower transition would most likely come from a smaller wavelength. An image of the visible light spectrum can be seen in Figure 3.

5. The energy difference between shells in the hydrogen atom can be calculated using the Rydberg formula:

$$\Delta E = -13.6eV(\frac{1}{n_f^2} - \frac{1}{n_i^2}) \tag{5}$$

The atom will get excited into these states through thermal energy (heat) or electromagnetic radiation. This energy that goes into the atom must remain conserved so the electron will jump to higher shell. Below is a table showing the energy associated with the different orders.

TABLE IV
ENERGY DIFFERENCES FOR VARIOUS COLOR ORDERS BASED ON EXPERIMENTAL $n_i$ VALUES.

| Color (Order) | Experimental $n_i$ | Energy Difference |
|---|---|---|
| Blue (1) | 5.003083 $\pm$ 0.091268 | -2.856 eV |
| Cyan (1) | 4.006698 $\pm$ 0.037084 | -2.522 eV |
| Red (1) | 3.002063 $\pm$ 0.008218 | -1.888 eV |
| Purple (2) | 6.006417 $\pm$ 0.079538 | -3.027 eV |
| Blue (2) | 5.001556 $\pm$ 0.040184 | -2.856 eV |
| Cyan (2) | 3.999338 $\pm$ 0.015553 | -2.522 eV |
| Purple (3) | 5.999825 $\pm$ 0.040569 | -3.027 eV |
| Red (2) | 2.999697 $\pm$ 0.002701 | -1.888 eV |

6. In this experiment due to the limitations of the equipment there would be no energy level that would ultimately yield a more precise measurement. But typically, the most precise measurements are found at higher energy transitions. These yield a more precise measurement due to wavelength corresponding to higher energy transition are often significantly more distinct and produce sharper well defined lines.

7. When observing the visible light from the spectrum of hydrogen, you typically see four lines, which are the transitions to the second energy level (n=2). These correspond to visible wavelengths (from red to violet). Other transitions, such as those from n=3 to n=1, fall outside the visible spectrum and are part of the ultraviolet or infrared ranges, which you cannot see with the naked eye. Below is an image of the visible light spectrum.

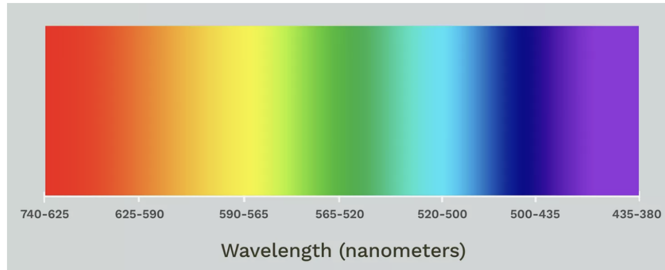


Fig. 3. Image depicting the visible light spectrum.

8. The $0^{th}$ order line appears in diffraction experiments and corresponds to the unshifted wavelength of the light being diffracted. It appears because the diffracted light at an angle of 0 degrees (directly along the optical axis) does not undergo any constructive or destructive interference. It is the light that has passed through the diffraction grating without being diffracted.

9. The way the uncertainties were measured was with the width of an individual line, as talked about in the analysis section. This was determined to 13 minutes, which is roughly between 0.01 and 0.015 percent of the final measurement. This precision is less accurate than other experiments in the course like the interferometer. From the interferometer is typically higher because the interferometer is designed to measure small wavelength differences with high accuracy. In contrast, spectroscopy often has slightly lower precision because of broader spectral lines and the challenges in detecting exact peaks in the spectrum.

10. The precision of the spectrometer was determined to 13 minutes, which is roughly between 0.01 and 0.015 percent of the final measurement. The precision of laser wavelength measurements in the last lab achieved higher precision. This higher precision is due to interferometers, in particular, being capable of achieving much more precise measurements of wavelength because they rely on the interference of light waves, which allows for very fine measurements of wavelength shifts.

11. Factors contributing to uncertainty are instrumental limitations, environmental factors, and human error. The resolution of the spectrometer, calibration errors, and noise from the detector can all contribute to uncertainty. For human error, misidentification of the spectral line position or improper alignment of the equipment can lead to random or systematic errors. Which was present in our experiment. This systematic error of not setting up the alignment proper caused a shift in all the angles measured during this experiment.

## VII. CONCLUSION

In this experiment, we successfully observed the spectrum of hydrogen by measuring the wavelengths of emitted light using a diffraction grating spectrometer. The resulting data allowed us to determine the quantum energy levels involved in the transitions, providing experimental validation for the

Rydberg and Bohr models of the hydrogen atom. Despite challenges with equipment alignment and measurement precision, our results closely aligned with theoretical predictions.